(In no particular order)

1. Calculate the fraction of the sky observable from any place on Earth at latitude 'l'. Note that you've to take into account the rotation of the Earth as well as its motion around the Sun.

2. Using spherical trignometry and assuming Earth's motion around the Sun to be uniform, calculate the number of days it takes for the Sun to go from the Vernal equinox to the zero shadow day in Bangalore.

3. Book: Radiative Processes in Astrophysics.
by
B. Rybicki & Alan P. Lightman

Prob: 1.1 1.9

4. Book: Carrol & Ostlie.
Prob: 9.3

5. By integrating the blackbody surface brightness over all frequencies and angles, derive the Stefan-Bolzmann Law $F = \sigma T^4$ and obtain

$$\sigma = \frac{2\pi^5 k_B^4}{15 c^2 h^3}$$

Also argue that the radiation constant 'a' in blackbody energy density $(u = aT^4)$ is given by

$$a = \frac{4\sigma}{c}$$

6. Book: Astrophysics in a nutshell
by
Dan Maoz

Chapter 1, Prob: 1 & 2

NB: You can ask help from tutors who can ask me in case of confusion

Note: This assignment contains problems as well as reading assignments.

① Some of the topics already discussed in the class can be found in chapter 3 of Maoz's book.

② Read section 3.3 of Maoz's book. Work through steps to derive the solar luminosity in Eq 3.53

③ Again, from Maoz's book, chap3, solve prob-1, prob-2, prob-3

④ Read through Chapter 8 of Bradley & Ostlie up to Saha ionisation formula. We will discuss it in next class.

- For star-formation and protostars read.
  Bradley & Ostlie, section 12.2 upto pg 432
- For various topics Chapter 10 of B&O

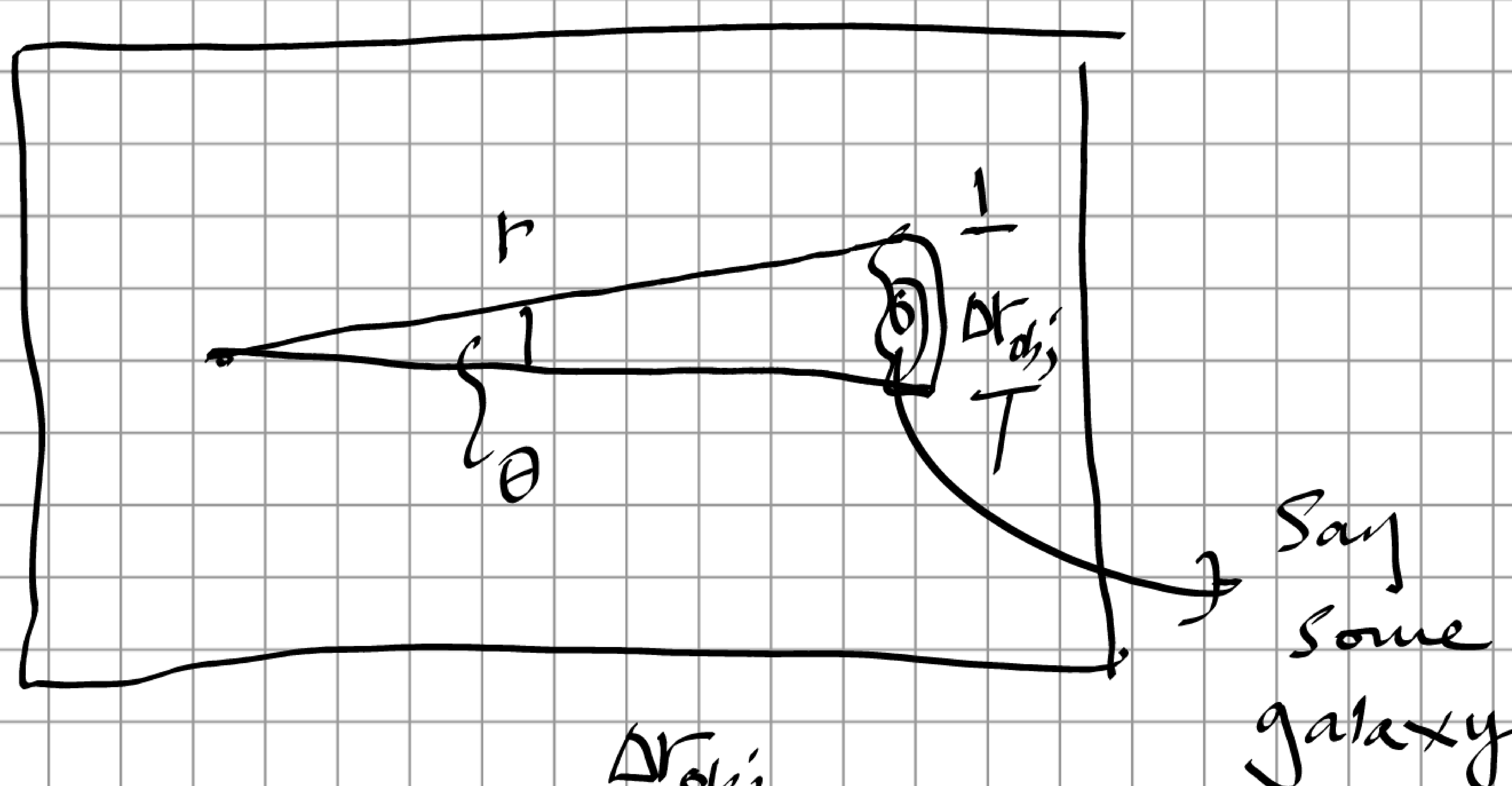
- Chapter 3 & 4 of Maoz. I have not covered supernovae & neutron stars, but from what was discussed in the class, it should all be straightforward.

Also, solve any problems from these chapters that seem accessible.

- **Angular diameter distance.**

as discussed in the class, bound objects don't expand with the universe. Therefore, we can measure angles in the comoving frame. (Angles don't change with uniform expansion)

Let Δr_{obj} be the comoving size of an object. and r be the comoving distance to it from us.



Then $\qquad \theta = \dfrac{\Delta r_{obj}}{r}$

Since the physical size of the galaxy coincides with the light path at the time 't' light left from the galaxy.

Physical size of galaxy $= l_p = a(t)\,\Delta r_p$

$$\therefore \quad \theta = \frac{l_p}{a(t)\,r}$$

$\Rightarrow$ Angular diameter distance

$$d_A = a(t)\,r$$

Noting that $\frac{a_0}{a} = 1+z$

$$d_A(z) = \frac{a_0\,r(z)}{1+z}$$

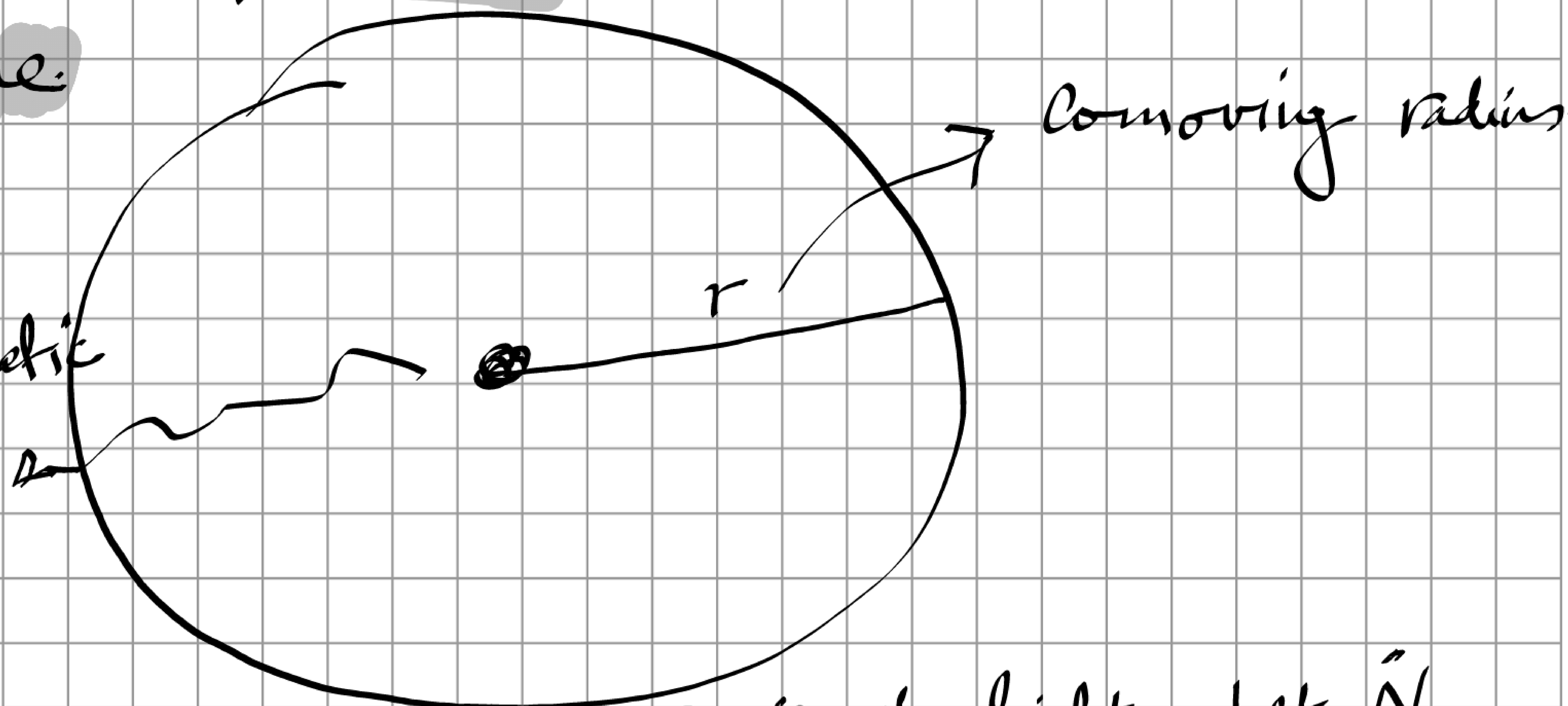
To be computed soon

Luminosity distance.

Imagine.

↓

Monochromatic Source of light



Assume a monochromatic source of light. Let $\dot{N}$ be the number of photons emitted in the frame of source. Then, the photon flux in the comoving frame is

$$\text{Photon flux} = \frac{\dot{N}}{4\pi r^2}$$

$\frac{dN}{dt}$ is evaluated at the source. Time runs differently at the source as compared to earth

Note that

$$ds^2 = c^2 dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + \underbrace{r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)}_{\text{this determines solid angles}}\right]$$

this determines solid angles

$$dA_{comoving} = r^2 d\Omega \xrightarrow{\text{integrate}} 4\pi r^2$$

$$\frac{dt \xrightarrow{} \text{source}}{dt_0 \searrow \text{earth}} = \frac{a(t)}{a_0} = \frac{1}{1+z}$$

[This is the same as the redshift formula]

Also, photons are redshifted and therefore have smaller energy.

$$\text{Obs. energy flux} = \frac{h\nu_0 \, dN/dt_0}{4\pi \underbrace{a_0^2 r^2}}$$

comoving to physical distance today $\to$ $\underbrace{a_0 r}$, Similarly all terms with subscript '0' are measured today.

$$= \left(\frac{\nu_0}{\nu}\right)\left(\frac{dt}{dt_0}\right) \frac{h\nu \, dN/dt}{4\pi \left(\frac{a_0}{a}\right)^2 d_A^2}$$

Source energy flux measured in the source frame

$$= \frac{L_0}{4\pi \, (1+z)^4 d_A^2} = \frac{L_0}{4\pi d_L^2}$$

$$d_L(z) = (1+z)^2 d_A$$

Derivation of $r(z)$

For light rays $ds = 0$. From FRW metric we obtain.

$$\frac{dr}{1-kr^2} = \frac{c\,dt}{a(t)}$$

We focus only on $k=0$ flat universe.

$$\therefore \quad dr = \pm \frac{c\,dt}{a(t)}$$

$$r(z) = \pm\, c \int \frac{dt}{a(t)}$$

If we are at $r=0$, the time decreases as we look at far away galaxies

To transfor RHS to an integration in z, note that

$$\frac{a_0}{a(t)} = 1 + z(t) \quad \leftarrow \text{definition.}$$

Diff. w.r.t. 't'

$$-\frac{a_0}{a^2(t)}\dot{a} = \frac{dz}{dt}$$

$$= -(1+z)\,H(z) = \frac{dz}{dt}$$

Placing the origin at our position

$$dr = -\frac{c\,dt}{a(t)} = -\frac{c}{a(x)}\frac{dt}{dz}\,dz$$

$$dr = \frac{c\,dz}{a(z)\,H(z)\,(1+z)}$$

Basically, the sign says that an object at 'r' away from us is seen in the past

$$r(z) = \int_0^z \frac{c\,dz'}{a(z')\,H(z')\,(1+z')}$$

or

$$a_0\,r(z) = c\int_0^z \frac{dz'}{H(z')}$$

Now we can write expressions for the two distances.

$$d_A(z) = \frac{c}{1+z}\int_0^z \frac{dz'}{H(z')}$$

$$d_L(z) = c\,(1+z)\int_0^z \frac{dz'}{H(z')}$$

To derive an expression for $H(z)$ recall the equation (Maoz, eq. 8.27)

$$\left(\frac{\dot{a}}{a}\right) = H^2 = \frac{8\pi G\rho}{3} - \frac{Kc^2}{a^2}$$

For a flat universe, $K=0$, Let us consider a universe with matter and radiation. Recall that the density equation

$$\dot{\rho} = -3H\left(\rho + \frac{P}{c^2}\right)^{*}$$

* (in the class we had used $c=1$)

For $P = w\rho c^2$

$$\rho(t) = \rho_0 (1+z)^{3(1+w)}$$

For two components $\rho = \rho_1 + \rho_2$
Again shifting to $c=1$, we have for matter $w=0$ and for radiation $w=1/3$, giving us.

$$\rho(z) = \rho_{om}(1+z)^3 + \rho_{oR}(1+z)^4$$

$$\therefore \quad H^2(z) = \frac{8\pi G}{3}\left[\rho_{om}(1+z)^3 + \rho_{OR}(1+z)^4\right]$$

Defining

$$h(z) = \frac{H(z)}{H_0}$$

$$h(z) = \frac{8\pi G}{3H_0^2}\left[\rho_{om}(1+z)^3 + \rho_{OR}(1+z)^4\right]$$

$$\rho_c \equiv \frac{8\pi G}{3H_0^2} \quad \left\{ \text{Define critical density} \right\}$$

and dimensionless

$$\Omega_{mo} \equiv \frac{\rho_{om}}{\rho_c}$$

$$\Omega_{RO} \equiv \frac{\rho_{OR}}{\rho_c}$$

$$h(z) = \Omega_{mo}(1+z)^3 + \Omega_{RO}(1+z)^4$$