Lecture - 14 (Varun Raghunathan)

Topics which will be covered

1) From classical to quantum communication (devices systems) → M. Fox, Quantum optics & lecture notes

2) Basics of Cryptography and Quantum key distribution.
↳ Neilsen & chung, Quantum computation Book

3) Quantum optical treatment light
↳ 2nd Quantization [Photon state $|n\rangle$, Coherent state $|\alpha\rangle$]
↳ quantization

4) understanding how quantum states interact with optical components (beam splitter + Interferometer)
Gerry and knight, intro to quantum optics.

Exam:- One in class exam - 3rd week of october. (1/3rd weightage)

Q * Why Quantum key distribution?

Ans
- most matured of the Quantum technologies
- Strategies application in Cryptography.
- ★ Photons are the carriers of quantum information?
  ↳ They tend to have long coherence time (advantage)
  ↳ its difficult to build quantum memories (disadvantage)
    ↳ interaction with matter qubits (very nascent field)

Q can we use long length of fiber (delay lines) to store memory?

Ans NO, delay line memory are prone to losses (drawback)

But we can do on the fly processing
This is possible when photon sources and detectors are efficient.

* Typical communication speeds → Room temp comm.

desirable (QKD Key rates) ≈ kbps - 10s of Mbps (wifi speed)

* we cant use RF or infrared have less energy the kT value is similar to $h\nu$ then it may thermalised. So not a good idea to use it.

Wavelength ranges used for quantum comm.

[1] visible: 400-700nm used for interfacing with atoms/ NV- centers / ion traps etc

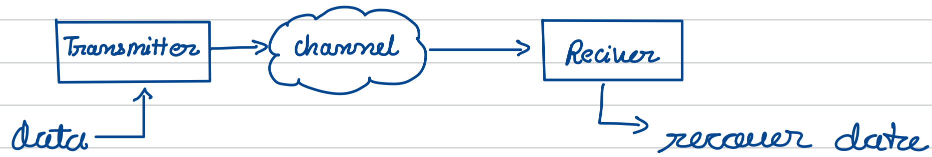
[2] shorter near IR: 700-1000nm → Free space communication.
[3] Near IR/TeleCom: 1300-1600nm

↓

Fiber-optic comm.
- leverage advancement telecom.
- InGaAs SPADs super conducting nanowire single photon detector (SNPD)

Free space communication:
- entangled photon sources (700-800nm range)
- Si-SPADs with efficiently
- needs strategies to mitigate sunlight interference

Classical Optical Communication System



Light sources: Bright photons (mean Photon no. is much much greater than one)
→ Laser
→ LED

Modulation: Encode data into optical carrier
↪ direct modulation (of current to a laser)
↪ indirect modulation (External modulator)
→ electron optic modulator.
→ modulate light

depending on state of light we choose accordingly.
- Intensity (IM) modulator
- Phase (PM) modulator
- frequency (FM) modulator
- Polarization modulators

Fibers
↪ Free space → in the context of QKD free-space is used typically for long distance communication (>1000 km)

$\alpha = 0.2\ \mathrm{dB/km}$
$L = 1000\ \mathrm{km}$

$$10\log\left(\frac{P_{out}}{P_{in}}\right) = 0.2\times10^3 = 200\ \mathrm{dB}$$

Detectors

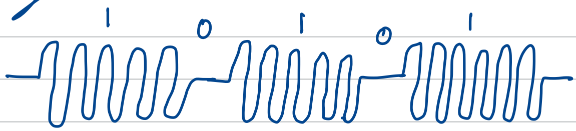
⇒ High speed photo detectors

↪ P-I-N diodes

↪ Avalanche photodetectors (photo diode with built in gain) → There are single photon detector based on it.

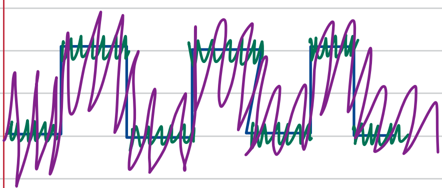
Noise and signal to Noise ratio (SNR)

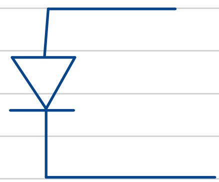
digital modulation onto the optical carrier



⇒ channel

⇓





— ⇒ 0 and 1 could be miss interpreted

— ⇒ little noise

Recovered digital signal with noise added

↪ Recover the signal at the output

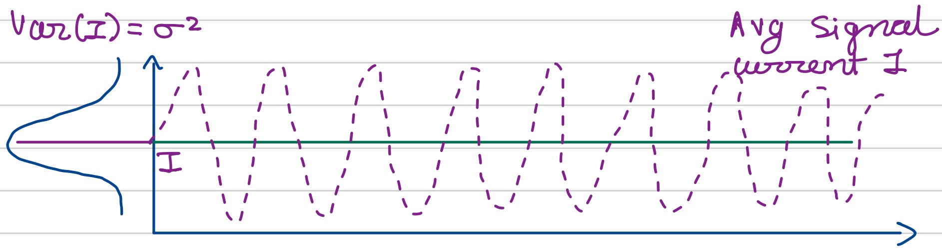
Noise can introduce error in the measurement.

Sources of Noise

* Shot noise ⇒ occurs because of the shot-to-shot (discretized) nature of photon quantization/absorption/detection. (quantum limited noise)

* Thermal noise ⇒ inherent noise due to the system (devices) operating at the temp (T).

- The way we quantify this is by using signal to noise ratio.



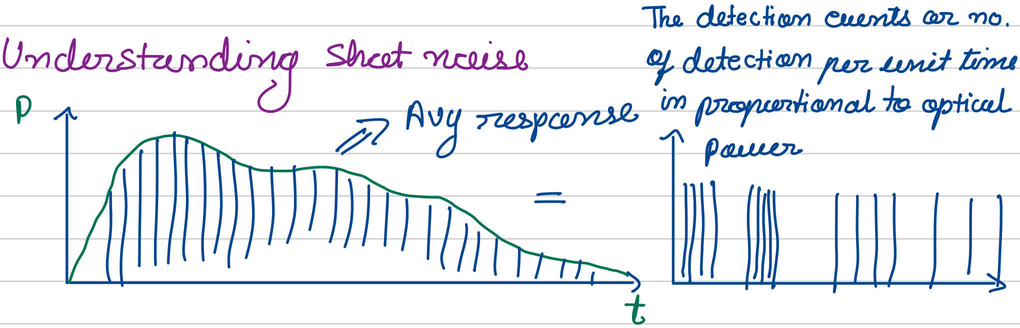
$$SNR = \frac{\text{mean}^2}{\text{Variance}} = \frac{\bar{I}^2}{\sigma^2} = \frac{\bar{I}^2}{\sigma_{shot}^2 + \sigma_{thermal}^2 + \dots\dots}$$

Noise power due to thermal noise $= 4kTB$

$$\sigma_{thermal}^2 = \frac{4kTB}{R} \qquad B = \text{Bandwidth}$$

$R \rightarrow$ Resistance

★ Understanding Shot noise



Think of a hypothetical parameter with very high temporal precision.

$$P = \frac{nh\nu}{T}$$

$n \equiv$ no. of photons in the measurement time window

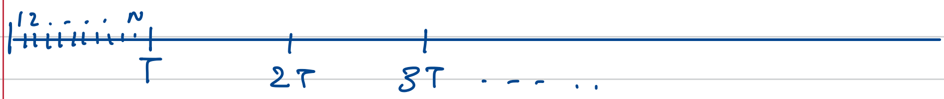
$T \equiv$ measurement in time window

$h\nu$ = energy of the Photon

Single photon detector + Time Tagger

what is the Prob of detecting n photon in a measurement time window T.

Consider the mean Photons no. $= \bar{n}$



divide the measurement time window in N subbins

Prob. of detecting the photon in a sub bin $p = \frac{\bar{n}}{N}$

$$P(n) = \lim_{n \to \infty} {}^{N}C_{n}\, p^{n} (1-p)^{N-n}$$

$$P(n) = \lim_{N \to \infty} {}^{N}C_{n}\, p^{n} (1-p)^{N-n}$$

$$\left.\begin{aligned} \mu(n \mid \text{shot noise limit}) &= \bar{n} \\ \sigma^{2}(n \mid \text{shot noise limit}) &= \bar{n} \end{aligned}\right\} \; \text{SNR}\big|_{\substack{\text{shot}\\\text{noise}\\\text{limit}}} = \frac{\bar{n}^{2}}{\bar{n}} = \bar{n}$$

current Variance due to shot noise

$$= \bar{n}(q/T)^{2} = (\bar{n} q/T)(q/T) = \bar{I} q (2B)$$

$$\boxed{\sigma^{2}_{\text{shot}} = 2q\bar{I}B} \quad \text{and} \quad \boxed{\sigma^{2}_{\text{Thermal}} = 4kTB/R}$$

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$$SNR = \frac{(\bar{I})^2}{\sigma_{shot}^2 + \sigma_{thermal}^2} = \frac{(R P_{opt})^2}{2q\bar{I}B + \frac{4kTB}{R}}$$

$R P_{opt} = I$

$R \rightarrow$ Responsitivity of the photo detector

detection without gain (PIN)

$$SNR|_{\text{with gain (APD)}} = \frac{(R P_{opt} G)^2}{2q(R P_{opt}) B G^2 F + \frac{4kTB}{R}}$$

gain in the current variance

Noise figure $\Rightarrow$ increase noise due to the gain process

G: gain from the APD

Recivers sensitivity

$SNR = SNR_0$

Recivers sensitivity refers to Popt required to achive $SNR = SNR_0$

$SNR(dB) = 10\log_{10}(SNR_0)$

we can come up with a power budget for the communication link ($P_{opt}|_{Rx}$ & $P_{opt}|_{Tx}$) to achive specific SNR performance

For fixed $P_{opt}|_{Tx}$ we can determine the max. length to support a requried SNR

We can come up with a power budget for the comm. link to achieve specific SNR performance or for fixed optical $P_{opt}|_{Tx}$ we can determine the max link length to support a marginal SNR.

SNR and BER are related   SNR↑ then BER↓

Bit error rate

In Quantum Key distribution we dont use SNR but QBER. (Quantum bit error rate)

Channel

↳ Fiber

↳ Free space

Impairment in the optical channel

Fiber channel: (1) optical losses (scattering & absorption)

(2) dispersion

(3) optical non linearity

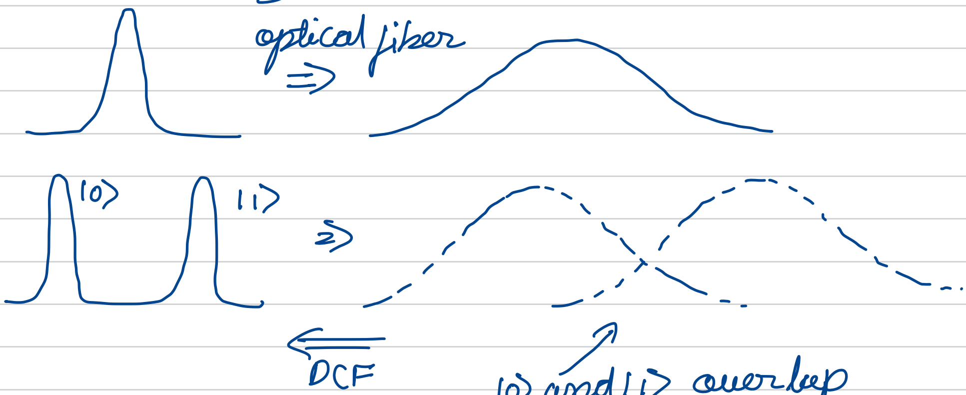
e.g

single photon

after propagation through a lossy channel

(easily become Possion distributed)

time encoding



* dispersion can increase the error probability

to eliminate we use dispersion compensation fiber (opposite n (Refractive index)

NON-linear Effects

* Intensity dependent optical response from the medium for optical nonlinearity.

common nonlinear - wave mixing

⇒ classical and quantum channel can have cross talk in four wave mixing.

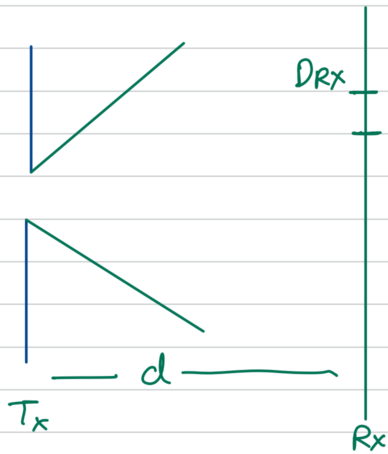
Raman Scattering

* Scattering of bright photons of the classical channel to the stokes and anti stoke band

If the quantum channel overlaps with the stokes and anti stokes band the quantum info can be compromised.

Free Space Channel

- Diffraction (beam spreading)
- Scattering due to atmosphere



diffraction

(path loss in comm.)

→ depends on aperture at R_x (Reciver)

→ also on wavelength.

* Friis formula for free space transmission

$$\frac{P_{Rx}}{P_{R_T}} \propto \frac{D_{Tx}^2}{\lambda^2}\frac{D_{Rx}^2}{d^2}$$

D_{Rx}, D_{Tx} aperture size

$d \rightarrow$ distance

$\lambda \rightarrow$ wavelength.

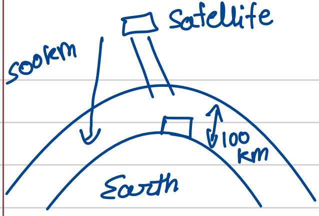
To minimise path losses :- $D_{Tx}, D_{Rx}\uparrow, \lambda\downarrow, d\downarrow$

* Scattering occurs due to difference in refraction index in the communication wavelength

Particle size relative to wavelength also influence the scattering.

Particle size $\simeq \lambda \Rightarrow$ Results in significant scattering.

Scattering results in loss of spatial coherence



overall path loss to Satellite band QKD to determine by the initial atmosphere scattering.

The losses for long distance FS-QKD is less when compared to fiber QKD

It is essential to have negligible beam tracking mechanism to ensure Tx-Rx are alinged for long distance communication.

For a fiber channel with 0.2 dB/km, 1000km fiber will result in 200 dB of loss

$$\frac{P_{Rx}}{P_{Tx}} = 10^{-20}$$

Long distance fiber comm. happen due to the use of optical amplifier..

- EDFA
- SOA
- Raman Amplification

No cloning theorm in quantum mechanics prevents the duplication of a general quantum state

⇓

For Quantum Comm. one cannot use optical amplifier

There's intrest in Quantum teleportation and quantum optics technology as a way to reproduce/ duplicate the quantum state.

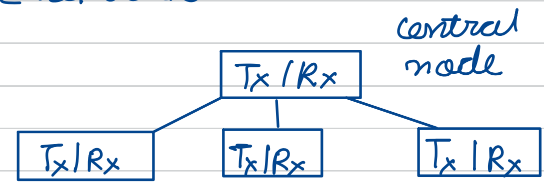
Different architecture for optical communication link.

Tx → Rx   Point to point link

Tx → Rx | Tx → Rx | Tx → Rx   Relay based point to point link

For QKD ⇒ Trusted Relay point to point link

Star network



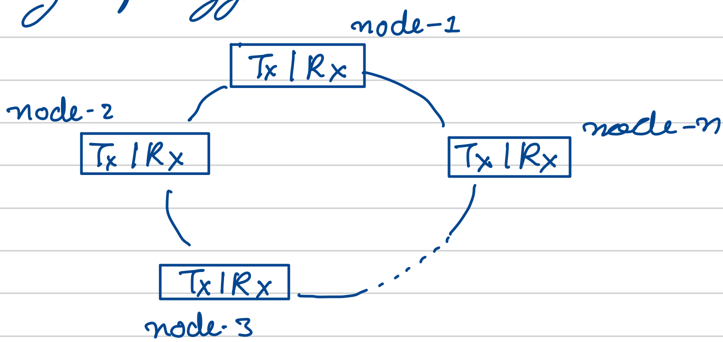
For QKD ⇒ entangled pair of photon at the central node get distributed to Alice - Bob (end to end)

MDI - QKD

measurement device independent QKD: states are prepared at the end node and transmitted to the central node for measurement.

measurement outcomes are known. who transmitted what is not known. that garuntees security.

Ring Topology



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RAS protocol

Alice	$\longleftrightarrow$	Bob
Eg. Bank e commerce site		Individual

Public key distributed to the individual
Encrypted message gets sent to the bank. $\Rightarrow$ Bank can decode the message with a private key

(Public and Private key)

Computational complexity $\rightarrow$ guarantees security for public key cryptography.

$\Rightarrow O(e^N)$ $N \rightarrow$ no. of digits which makes up $n = p \times q$

$E(M) = M^e \,(\text{mod}.\, n)$

$D(E|M) = E(M)^d \,(\text{mod}.\, n)$

* Quantum key distribution

- its based on principle of QM with rigorous security proof to guarantee security

* Qubit states
  For eg:- H and V pol of photon in a single mode

* Entangled photon
  consider two modes a & b, $|0\rangle_a, |1\rangle_a$ & $|0\rangle_b, |1\rangle_b$

suppose $\Rightarrow$ $|\psi\rangle_{ab} = \dfrac{|0_a 0_b\rangle + |1_a 1_b\rangle}{\sqrt{2}}$ (just one Ex)

★ Polarization states used to encode quantum information

$\{|H\rangle, |V\rangle\} \rightarrow$ Rectilinear Basis

$\{|D\rangle, |A\rangle\} \rightarrow$ diagonal Basis

$\{|L\rangle, |R\rangle\} \rightarrow$ circular Basis

Pol. bases set's which are not orthogonal to other set.

$|D\rangle = \dfrac{|H\rangle + |V\rangle}{\sqrt{2}}$ → diagonal

$|A\rangle = \dfrac{|H\rangle - |V\rangle}{\sqrt{2}}$ → Antidiagonal

$|L\rangle = \dfrac{|H\rangle + i|V\rangle}{\sqrt{2}}$ → left circular

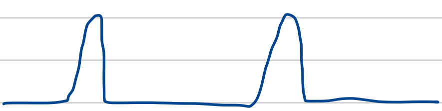
$|R\rangle = \dfrac{|H\rangle - i|V\rangle}{\sqrt{2}}$ → Right circular

wavelength is a degree of freedom:

$\{|1\rangle_\lambda, |0\rangle_\lambda\}$

$\{|1\rangle_\lambda, |0\rangle_\lambda\}$

optical pulse in time can be used to encode quantum states:



$|E\rangle$ $\quad$ $|L\rangle$

Early, Late

$\{|E\rangle, |L\rangle\}$

$\{\alpha|E\rangle, \beta e^{i\theta}|L\rangle\}$

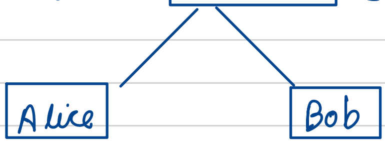
90° or 270° phase delay (might)

- when we do this in time we call it Time-bin encoding → suitable for multi dimensional encoding
- Encoding in special modes of fiber or free space

$\{|M_1\rangle, |M_2\rangle, \ldots, |M_n\rangle\}$

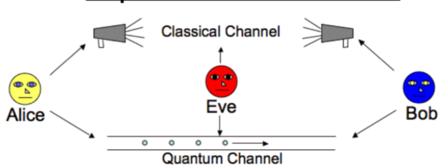
- Orbital angular momentum states are also used for multi-dimensional encoding

* in slide → $\boxed{\text{Charlie}}$ entangled photon source



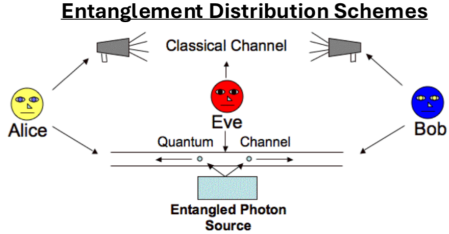
Quantum Communication System - Classification

Prepare and Measure Schemes



- Alice prepares states and sends this to Bob who does the measurement
- Forward or reverse reconciliation (to come up with a common key)
- Eve attempts to tap into the communication to gain some/ all the information
- During post-processing the extent of Eavesdropping is estimated to key or reject the block of communication
- Example: BB84, COW, T12 etc.

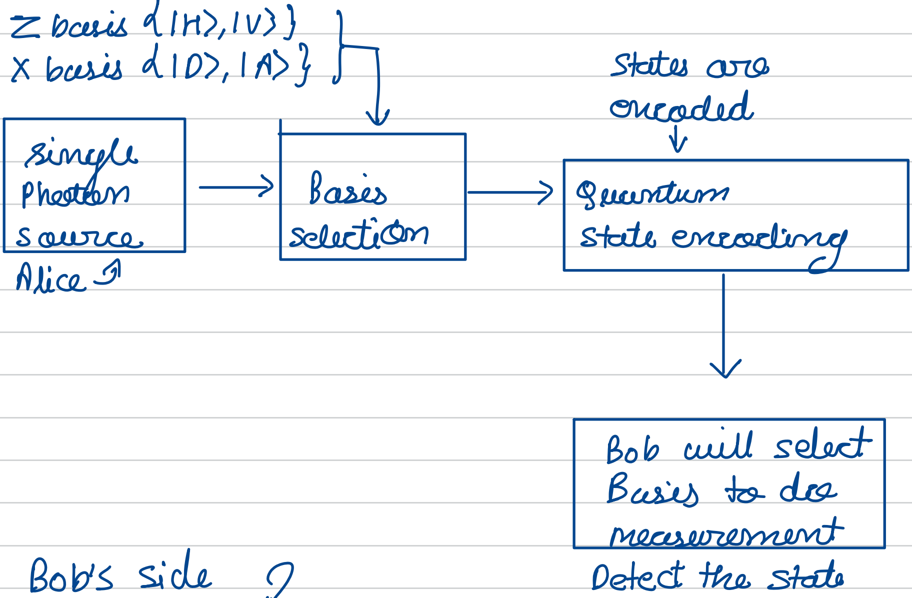
Entanglement Distribution Schemes



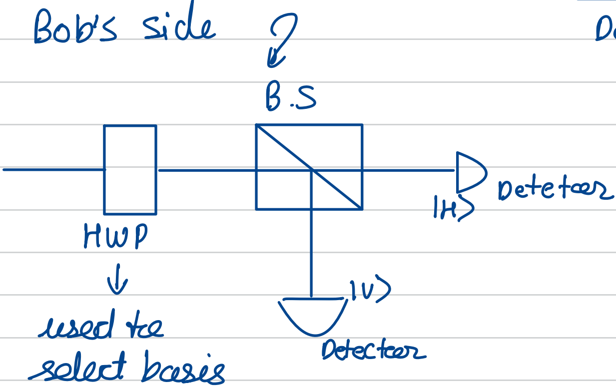
- A central party (can be Eve as well) generates entangled pairs of photons
- Entangled photons distributed to Alice and Bob
- Based on their measurement they know what the other persons state is
- During post-processing the strength of entanglement is estimated using suitable metric (e.g. CHSH inequality)
- Amenable to repeaters, entanglement swapping etc. → future quantum internet
- Example: E91, BBM92 etc.

Consider an entangled photon source at the central node

$$|\psi^+_{ab}\rangle = \frac{|0_a 0_b\rangle + |1_a 1_b\rangle}{\sqrt{2}}$$



Bob's side



* Original BB84 protocol proposed the use of:-

- ideal single photon source
- ideal basis/ quantum states encodes
  ideal basis decode

- ideal single photon electron

$\Downarrow$

in reality the above components end up being non ideal

↳ implementation assumption made

↳ This opens side channel for attacks

<u>Note</u>

★ Private key's are as long as the message

Lecture - 17

Exam on 17th Oct - 10 AM

Make up lec on 10th - 10AM

Quantum light source → Encoding quantisation → Channel → Quantum state manipulation → detection

Quantum light source:

- single photon source
- weak coherent source (attenuated laser source)
- Entangled photon pair source

Encoding quantisation: modulation to encode quantum states

↳ Electro-optic modulation

Channel: Fiber, Freespace

Quantum state manipulation:

- Passive
  - Beam splitter
  - Interferometer
- Active
  - Electrooptic modulator

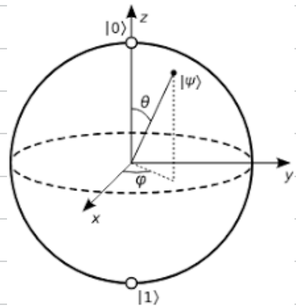
detection:

- Single photon detector
  - ↳ SPAD
  - ↳ SCSPD

★ Also Note :- laser is not a single photon source even when we attenuate it gives poissons distribution.

Represent quantum states in Bloch Sphere

$\{0, 1\}$



$x : \left\{ \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right\}$

$y : \left\{ \frac{|0\rangle + i|1\rangle}{\sqrt{2}}, \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \right\}$

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi} \sin\left(\frac{\theta}{2}\right)|1\rangle$$

$|0\rangle$ and $|1\rangle$ in light can represent polarisation and time. (one of the most imp)

Single Photon Source

(1) (Ref:- Q optic Mark Fox ch-5)

Source which emits deterministically one photon for every excitation

laser is not a single photon state even if we attenuate

If we can restric dos (density of state) available through an impulse junction then only state is available for excitation the population (e^- laton) and emit one photon

⇒ eg Quantum dots, Diamond-Nitrogen Vacancy center monolyer of 2D material.

- in bulk there are so many state but if we restric then we can get.

- often single photon emission is studied at low temp

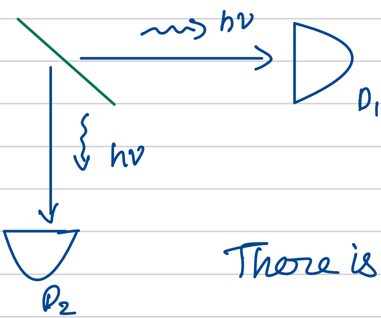
How do you represent these states?

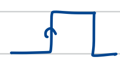
Single photon states more generally are represented as Fock state (photon state $\Rightarrow |n\rangle$)

$|0\rangle \Rightarrow$ Vaccum state
$|1\rangle \Rightarrow$ single photon state
$|2\rangle \Rightarrow$ two photon state

emits multiple photon in one packet at a time

Q.D in active region of PN Junction.
(quality depends on the homogenity and hetrogenity of the PN Junction)

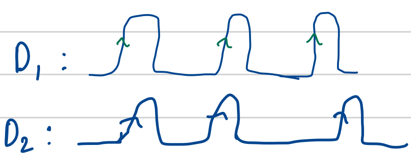


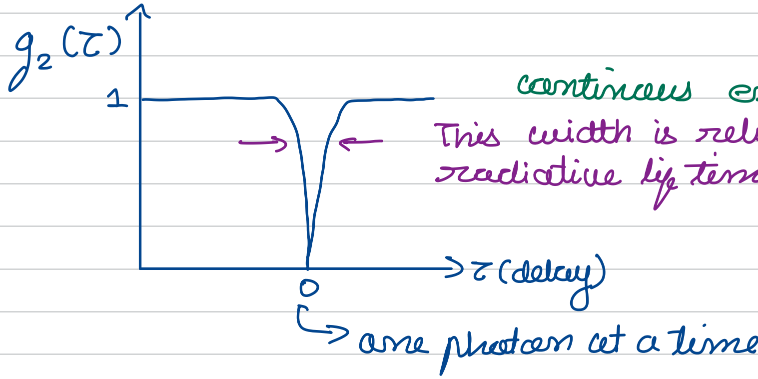


Coincidence measurement is done

There is a metric called g_2.

$$g_2(\tau) = \frac{\langle n_1(t)\, n_2(t-\tau)\rangle}{\langle n_1(t_1)\rangle \langle n_1(t-\tau)\rangle}$$



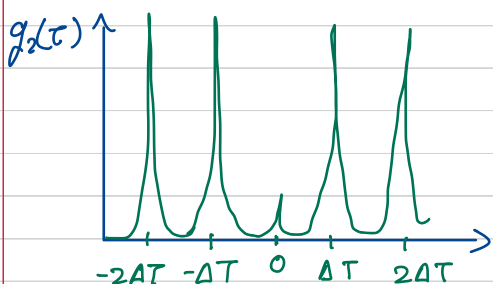


continuous excitation

This width is related to the radiative life time.

$g_2(0)$ should be as close to zero for a good single photon source

other, $g_2(0) < 0.5$ has single photon properties.

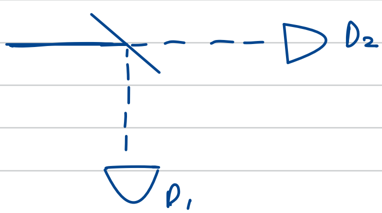


Pulsed excitation

Popular example of this (SPE) is Random photon generator.

$$|\psi_{12}\rangle = \frac{|1_1 0_2\rangle + |0_1 1_2\rangle}{\sqrt{2}}$$

Path entangled single photon state.



(2) Entangled Photon pair sources:

A multipartite state is said to be entangled if it cannot be factorized into its individual states

Bipartite entangled states between mode-1 & mode-2.

Bell states

$$|\phi^{\pm}\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}}$$

$$|\psi^{\pm}\rangle = \frac{|01\rangle \pm |10\rangle}{\sqrt{2}}$$

If there is 50% of measuring the 1st part and 50% chance of the 2nd part these are maximally entangle state

* consider a bipartite system made of:

$\{|0\rangle_1, |1\rangle_1\}$ & $\{|0\rangle_2, |1\rangle_1\}$

any general state of the bipartite states can be written as:

$$(a_1|0\rangle + b_1|1\rangle) \otimes (a_2|0\rangle_2 + b_2|1\rangle_2)$$

$$= a_1a_2|0_1 0_2\rangle + b_1b_2|1_1 1_2\rangle + a_1b_2|0_1 1_2\rangle + a_2b_1|1_1 0_2\rangle$$

consider: $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|0_1 0_2\rangle + |1_1 1_2\rangle)$

for $|\phi^+\rangle$ to be a general state of modes 1 & 2:

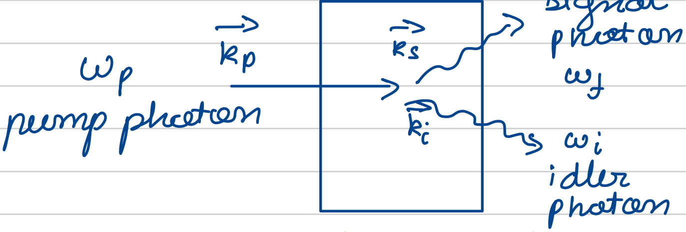
$a_1 a_2 = 1/\sqrt{2}$ $\qquad$ $b_1 b_2 = 1/\sqrt{2}$

$a_1 b_2 = 0$ $\qquad$ $a_2 b_1 = 0$

This is not satisfied for valid values of a_2, a_1, b_1, b_2.

$\Rightarrow |\phi_+\rangle =$ cannot be separated to its individual state.

To simultaneous generate two photon with strong correlation we make use of spontaneous parametric down conversion (SPDC)



Reverse process should also hold

also consider phase or wave vector matching

Spontaneity splitting of one pump photon to a signal and idler photon

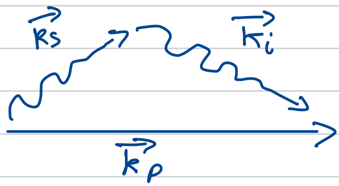
For particular process:

Energy conversation among the photon is required.

I) $\omega_p = \omega_s + \omega_i$

II) wave vector matching or phase matching

$$\vec{k}_p = \vec{k}_s + \vec{k}_i$$



Phase matching is achieved for one combination of p, s, i such that the direction of the photon and energy of photon are well defined

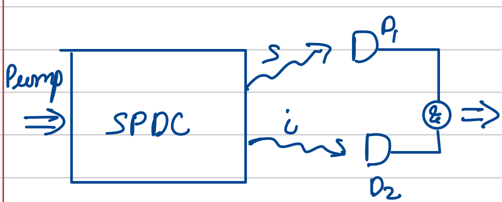
$\omega_s = \omega_i \Rightarrow$ degenerate SPDC process

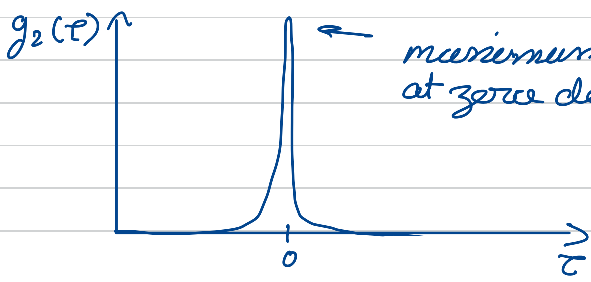
$$\omega_p \longrightarrow \frac{\omega_p}{2} + \frac{\omega_p}{2}$$

(they are correlated in E, t, wave vector)

$\omega_s \neq \omega_i \Rightarrow$ Non degenerate SPDC process

The signal and idler photon are highly correlated in energy, direction (wave vector), time, polarisation (due to phase matching of the signal)





maximum correlated at zero delay.

Wave vector mis match:

$$\Delta \vec{k} = \vec{k_p} - (\vec{k_s} + \vec{k_i})$$

$$= n_p \cdot \frac{2\pi}{\lambda_p} \hat{e}_p - n_s \frac{2\pi}{\lambda} \hat{e}_s - n_i \frac{2\pi}{\lambda_i} \hat{e}_i$$

$$= 2\pi \left[\frac{n_p}{\lambda_p} \hat{e}_p - \frac{n_s}{\lambda_s} \hat{e}_s - \frac{n_i}{\lambda_i} \hat{e}_i \right]$$

Consider the copropagation of the wave in the crystal:

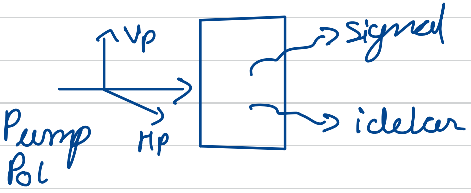
$$\Delta \vec{k} = 2\pi \hat{e}_p \left[\frac{n_p}{\lambda_p} - \frac{n_s}{\lambda_s} - \frac{n_i}{\lambda_i} \right]$$

Energy conservation require:

$$\frac{1}{\lambda_p} = \frac{1}{\lambda_s} + \frac{1}{\lambda_i}$$

Birefringence properties of the crystal are used to satisfy the phase matching condition.

To access birefringence properties of the crystal, different polarisation and crystal nature are combined.



Different Types of birefringent phase velocity

1) Type-I signal and Idler are of the same polarisation orthogonal to pump.

$$\left.\begin{array}{l} H_p \rightarrow V_s V_i \\ V_p \rightarrow H_s H_i \end{array}\right\}$$

2) Type-II: signal and Idler are of orthogonal polarisation

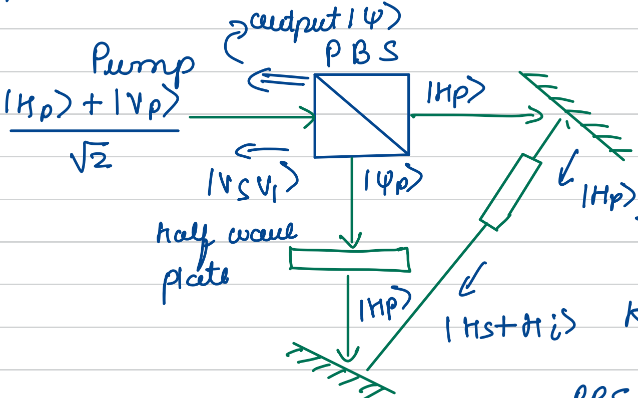
$$H_p \text{ or } V_p \longrightarrow H_s V_i \text{ or } V_s H_i$$

3) Type 0: these polarisation are identical

$$H_p \rightarrow H_s, H_i \quad \text{or} \quad V_p \rightarrow V_s, V_i$$

⇒ Note: Not all types of phase matching or polarisation combination will be satisfied.

Sagnac loop



PBS → Polarisation Beam Splitter.

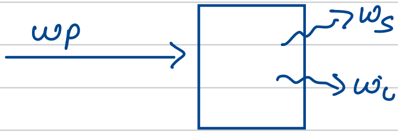
At the output (Pump input port)

$$|\psi\rangle = \frac{|V_s V_i\rangle + |H_s H_i\rangle}{\sqrt{2}}$$

check for type-I, where the output is measured.

Lecture-18

Entangled Photon sources (continued)



Spontaneous parametric down conversion

Non linear optical crystal

(1) Energy conservation: $\omega_p = \omega_s + \omega_i$

(2) phase matching : $\vec{k_p} = \vec{k_s} + \vec{k_i}$

(wave vector matching)

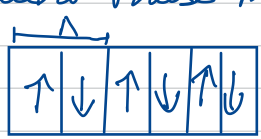
Technique to architeture phase matching

1) Birefringent phase matching

↳ Anisotropic crystal & choose appropriate pol.

can result in beam walk off between P, S, i

2) Quasi Phase matching



$\Delta k = k_p - k_s - k_i + q \frac{2\pi}{\Lambda} = 0$

⇒ poled domains

(PP) Periodically poled crystal to achive phase matching

e.g:- PP-LN
PP-KTP etc.

crystal axes is not rotated to achieve phase matching $\omega_p\ \omega_s\ \omega_i$ remain collinear

3) wave guide → Dispersion phase matching
e.g LN wave guide.

Types of Phase matching

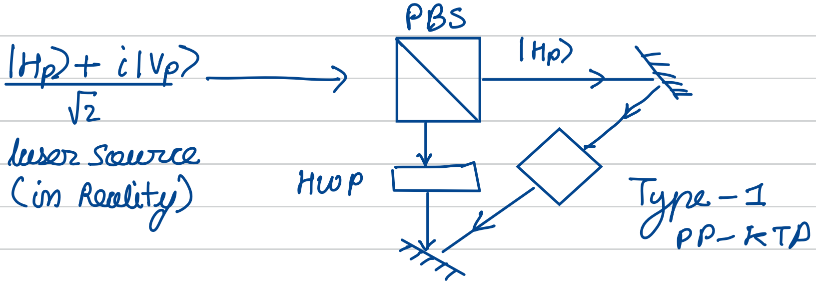
Type-0: All 3 pol. states are identical

Type-1: Pol. of the pump is orthogonal to the signal and idler.

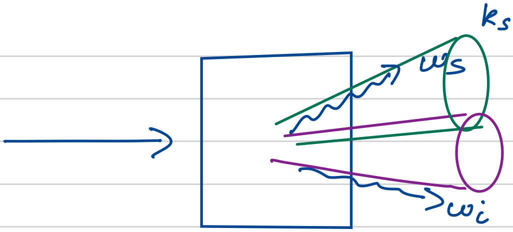
Type-2: S & i are orthogonal pol.

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left[|H_s H_i\rangle \pm |V_s V_i\rangle\right] \Rightarrow$$ Type-0 or Type-1 SPDC process

$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left[|H_s V_i\rangle \pm |V_s H_i\rangle\right] \Rightarrow$$ Type-2 SPDC process



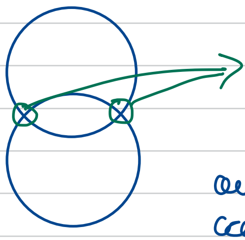
$$\frac{|H_s H_i\rangle + i|V_s V_i\rangle}{\sqrt{2}}$$



signal emission cone

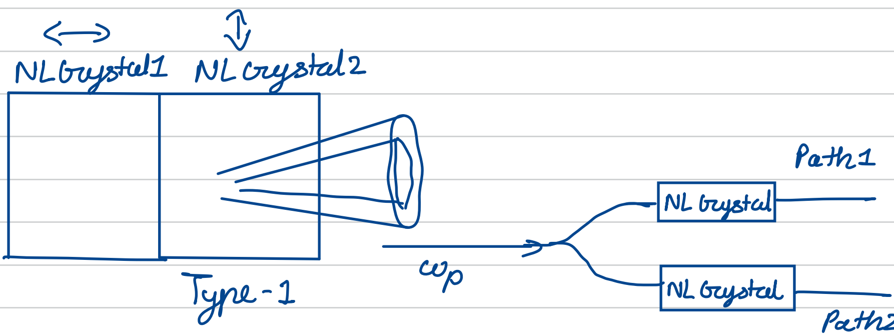
idler emission cone

Type-2 SPDC process



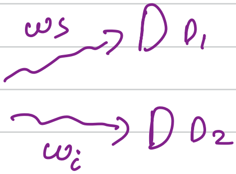
$$\frac{|H_s V_i\rangle + |V_s H_i\rangle}{\sqrt{2}}$$

overlap points of the two emission cone corresponds to the entangled photon.

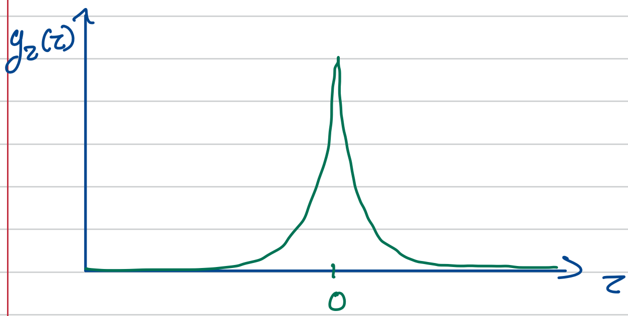


$$\frac{|10\rangle + |01\rangle}{\sqrt{2}}$$

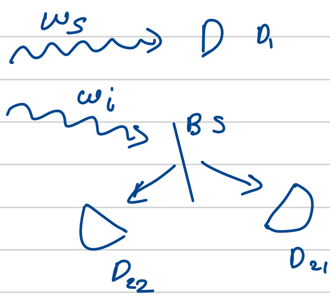
$|1_{s_1} 1_{i_1} 0_{s_2} 0_{i_2}\rangle + |0_{s_1} 0_{i_1} 1_{s_2} 1_{i_2}\rangle$ these States could also be Created.



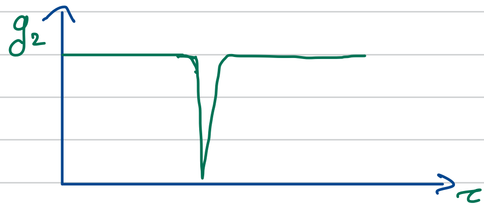
$$g_2(\tau) = \frac{\langle n_1(t)\, n_2(t-\tau)\rangle}{\langle n_1(t)\rangle \langle n_2(t-\tau)\rangle}$$



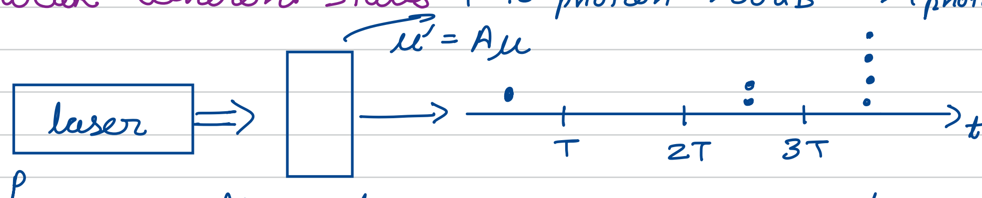
Heralde g_2 measurement



if ω_s get deflected what are the properties of ω_i?



Weak coherent states (10^6 photon $\rightarrow$ 60 dB $\rightarrow$ 1 photon



Attenuator reduce mean photon

on an average get 1 or <1 photon per measurement time window

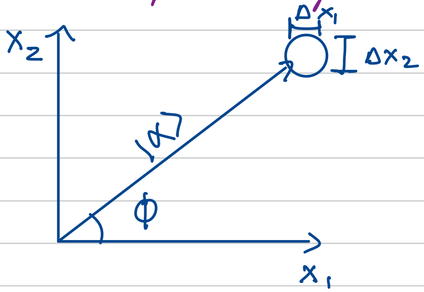
$$\mu = \frac{PT}{h\nu}$$

* Laser source are modelled as coherent states is quantum optics.

$|\alpha\rangle \equiv$ classical picture $|\alpha| e^{i\phi}$

in quadrature $\Rightarrow x_1 + i x_2$

Phase Space representation of $\alpha = x_1 + i x_2$



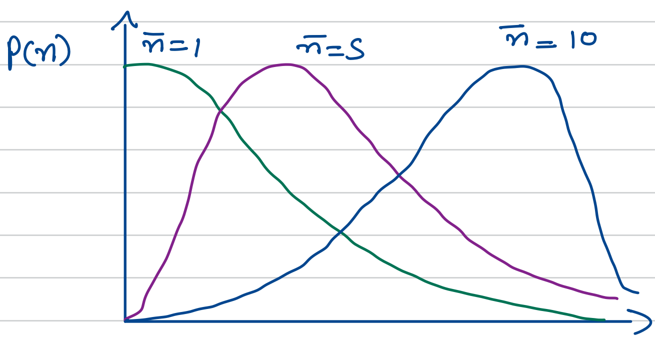
Short noise linked variation in α

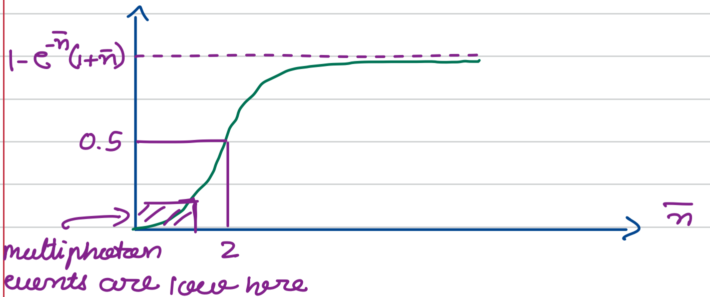
distribution of Photon follows Poisson distribution

$$P(n) = \frac{e^{-\bar{n}}\,\bar{n}^{n}}{n!}, \quad \bar{n} \text{ is mean no. of photon}$$

Probability of having ≥ 1 photon

$$= 1 - P(0) - P(1) = 1 - e^{-\bar{n}} - e^{-\bar{n}}\bar{n} = 1 - e^{-\bar{n}}(1+\bar{n})$$





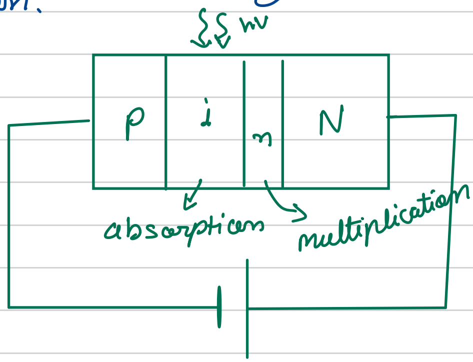
Search:- Squeezed States

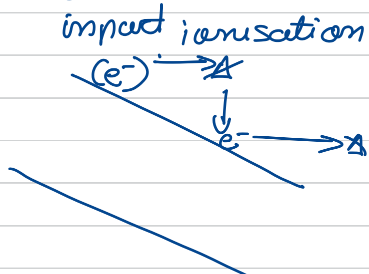
- Multiphoton events result in photon no. splitting attacks.

Single Photon detector

* SPAD ⇒ Semiconductor photo diode which work on the principle of Avalanch multiplication

* SNSPD ⇒ Any detection with built in gain is suitable for making suitable single photon detection.





avalench process
(multiplication process)

- e^- as they drift under the influence of the applied reverse bias they avalench due to the large E-field at reverse bias.

- Impact ionization results in ⇒ $1e^- \rightarrow 2e^-$
  1 hole

To keep the multiplication process under control 1 carrier (e^- hole) is preferentially multipled.

⇓

Good Avalanche electrons are chosen for single photon detection applicable.

After Pulsing

Unwanted Pulses of current detected after the photon detection has happend.

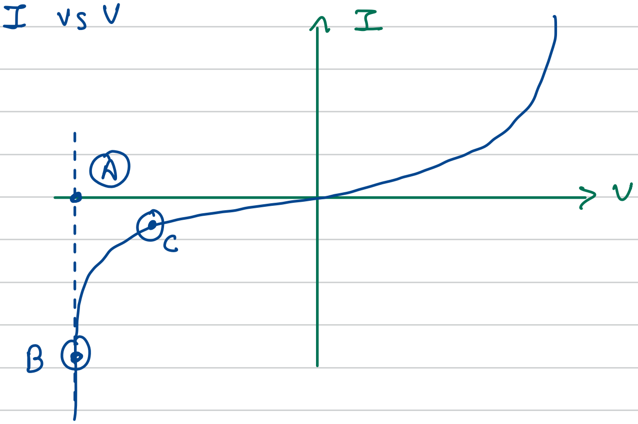
(Creates echo kind of signal) $\Downarrow$

The way this is prevented, in the circuit a latching mechanism dis-arm the detector

$\Downarrow$

The time for this is called the dead time.

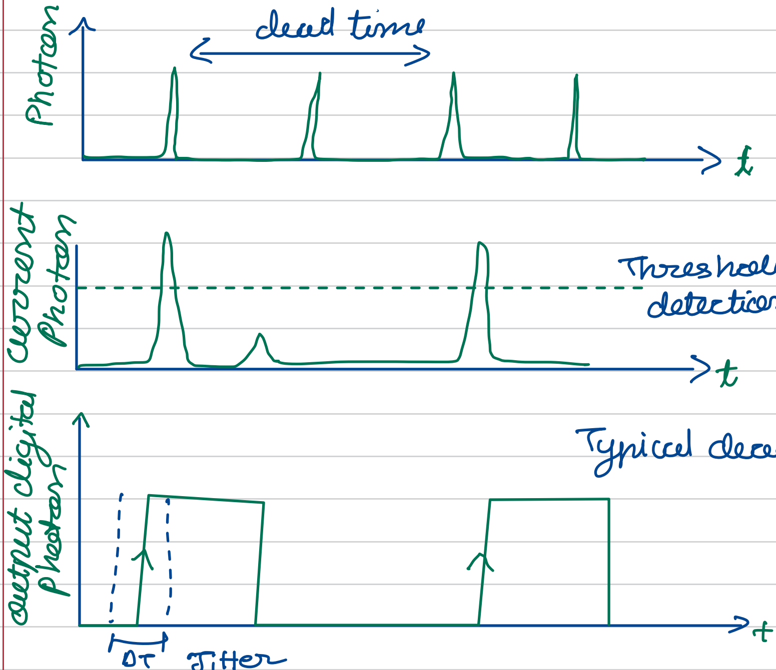
★ I vs V



A $\longrightarrow$ bias pt of the detector
B $\longrightarrow$ pt. where current pulse is detected
C $\longrightarrow$ latching pt to turn off the multiplication

Other than light there can be multiplication of thermally excited carriers (dark current)

Geiger Counting Mode



Typical dead time:
10 sec of μsec

TDC (Time for digital counter)

1 photon $\xrightarrow{?}$ 1 e^-/hole pair

efficiency of the detector (η)

$\Downarrow$

Equivalent to the probability of detection.

For SPADs:

* Si - SPAD (400 - 1000 nm wavelength)
  $\eta > 50\%$

* InGaAs SPADs (800-2000nm wavelength)

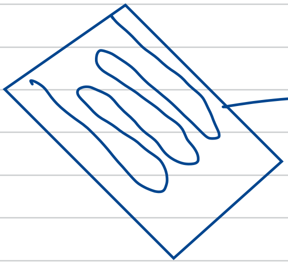
$\eta \approx 10-30\%$

* Jitter 50-200 psec

↪ large Jitter → problem

Super Conducting nanowire (SPD):
The state of the detector is changed from superconducting to insulting with incident photon

(Transition edge sensor are variants of SNSPD)



• Meandering nanowire of superconducting material

• NbN NbTiN WSi MoSi
Typical $T_{nc} \approx 10K$

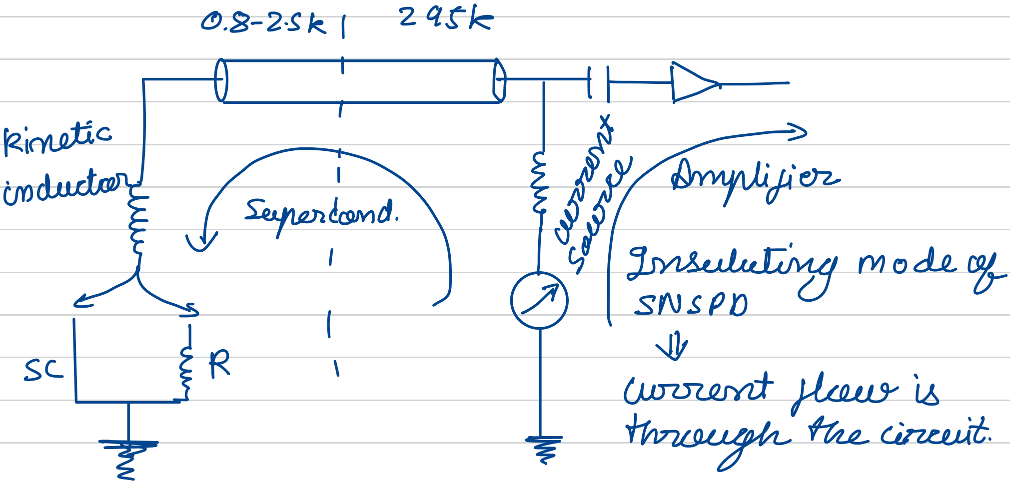
• Operating term of SNSPD:

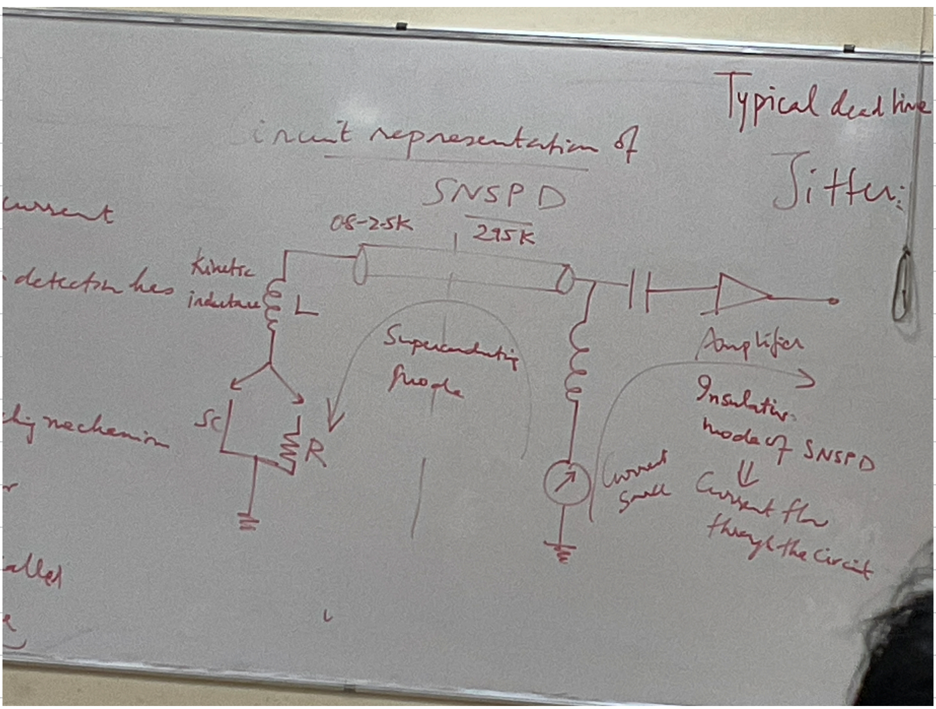
0.8 to 2k

Incident photon break the cooper pair there by disrupting the superconductivity.

After a finite time the SNSPD gets back to superconducting state.

Circuit representation of SNSPD



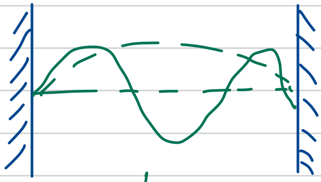


Some Typical performance metrits of

1. High efficiency (>90%)
2. Low dark counts (1-100 counts)
3. Jitter (<10ps)
4. Short dead time (40-100ns)
5. wide wavelength operation (visible/near IR/mid IR)

Lecture - 19

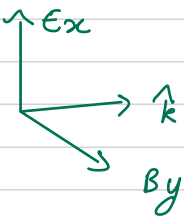
Reference (line to line) — Gerry and Knight Chp-2



optical cavity

$\omega_m = c \, \frac{m\pi}{L}$
$\downarrow$
Resonant frequency

Consider TEM plane wave coupled into



$$E_x(z,t) = \left(\frac{2\omega^2}{V\epsilon_0}\right) q(t) \sin(kz)$$

$\omega \to$ freq
$V \to$ volume of cavity
$\epsilon_0 \to$ free space permeability

Making use of $\nabla \times B = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$ substituting E_y in the RHS and solving for B

$$B_y = \left(\frac{2\omega^2}{V\epsilon_0}\right)^{1/2} \left(\frac{\mu_0 \epsilon_0}{k}\right) \dot{q}(t) \cos(kz)$$

$q(t) \to$ This is equivalent to the position variable in a harmonic osc.

Total energy of the TEM wave = Energy stored in E field

+ Energy in B field

$$H = \frac{1}{2}\int dv \left[\varepsilon_0 E^2(r,t) + \frac{1}{\mu_0} B^2(r,t) \right]$$

$$= \frac{1}{2}\int dv \left[\varepsilon_0 E_x^2(z,t) + \frac{1}{\mu_0} B_y^2(z,t) \right]$$

$$H = \frac{1}{2}(p^2 + \omega^2 q^2)$$

Replacing measurable variable by operators

$$\hat{E}_x = \left(\frac{2\omega^2}{V\varepsilon_0}\right)^{1/2} \hat{q} \sin(kz)$$

$$\hat{B}_y = \left(\frac{2\omega^2}{V\varepsilon_0}\right)^{1/2} \left(\frac{\mu_0 \varepsilon_0}{k}\right) \hat{p} \cos kz$$

$$[\hat{q}, \hat{p}] = i\hbar$$

$$\hat{H} = \frac{1}{2}\left[\omega^2 \hat{q}^2 + \hat{p}^2\right]$$

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

dirac's formalism of the ladder operator

$$\hat{a} = \frac{1}{\sqrt{2\hbar\omega}}(\omega\hat{q} + i\hat{p}) \qquad \hat{q} = \sqrt{\frac{\hbar}{2\omega}}(\hat{a} + \hat{a}^+)$$

$$\hat{a}^+ = \frac{1}{\sqrt{2\hbar\omega}}(\omega\hat{q} - i\hat{p}) \qquad \hat{p} = \frac{1}{i}\sqrt{\frac{\hbar\omega}{2}}(\hat{a} - \hat{a}^+)$$

$$\hat{E}_x = E_0 (\hat{a} + \hat{a}^+) \sin kz \qquad E_0 = \sqrt{\frac{\hbar\omega}{\varepsilon_0 V}}$$

$$\hat{B}_y = \frac{B_0}{i} (\hat{a} - \hat{a}^+) \cos kz \qquad B_0 = \frac{\mu_0}{k} \sqrt{\frac{\varepsilon_0 \hbar \omega^3}{V}}$$

the electric field comp. corresponds to 1 photon energy

Magnetic field amplitude correspond to 1 photon energy

$$\hat{H} = \hbar\omega \left(\hat{a}^+ a + \frac{1}{2}\right)$$

$$[\hat{q}, \hat{p}] = i\hbar \Rightarrow [\hat{a}, \hat{a}^+] = 1$$

$$\left[\sqrt{\frac{\hbar}{2\omega}} (\hat{a} + \hat{a}^+), \frac{1}{i} (\hat{a} - \hat{a}^+)\right] = i\hbar$$

$$\frac{1}{2i} \left[\underbrace{\cancel{[\hat{a}, \hat{a}]}}_{=0} - [\hat{a}^+ \hat{a}^+] - [\hat{a}\, \hat{a}^+] + [\hat{a}^+, a]\right] = i$$

$$\hat{H} |\psi\rangle = E |\psi\rangle$$

$$\hat{H} |n\rangle = E_n |n\rangle$$ $\Rightarrow$ energy eigen vaule equation in terms of the eigenvalue E_n and eigenfunction $|n\rangle$.

$$\hbar\omega (\underbrace{\hat{a}^+ \hat{a}}_{\text{Number operator}} + 1/2) |n\rangle = E_n |n\rangle$$

$\hat{N} = \hat{a}^{+}\hat{a} \Rightarrow$ No. of photon in the state

$\langle n | \hat{a}^{+}\hat{a} | n \rangle = n$

$\langle n | H | n \rangle = \hbar\omega(n + 1/2)$

Heisenberg Equation: $\dfrac{d\hat{O}}{dt} = \dfrac{i}{\hbar}[\hat{H}, \hat{O}]$

For $\hat{a}$: $\dfrac{d\hat{a}}{dt} = \dfrac{i}{\hbar}[\hbar\omega(\hat{a}^{+}\hat{a} + 1/2), \hat{a}]$

$$= \frac{i}{\hbar}\hbar\omega[\hat{a}^{+}\hat{a}, \hat{a}]$$

$$= -i\omega\hat{a}$$

$\hat{a}(t) = \hat{a}(0)e^{-i\omega t}$ similarly; $\hat{a}^{+}(t) = a^{+}(0)e^{i\omega t}$

$$\hat{E}_x(z,t) = E_0(\hat{a}^+ e^{i\omega t} + \hat{a} e^{-i\omega t}) \sin kz$$

$$\hat{H}|n\rangle = E_n|n\rangle$$

$$\hbar\omega(\hat{a}^+\hat{a} + 1/2)|n\rangle = E_n|n\rangle$$

applying $\hat{a}$ to both side

$$[\hat{a}, \hat{a}^+] = 1$$

$$\hbar\omega(\hat{a}\hat{a}^+\hat{a} + \frac{\hat{a}}{2})|n\rangle = E_n \hat{a}|n\rangle$$

$$\hbar\omega(\hat{a}\hat{a}^+ + \frac{1}{2})\hat{a}|n\rangle = E_n \hat{a}|n\rangle$$

$$\hbar\omega(1 + \hat{a}^+\hat{a} + 1/2)\hat{a}|n\rangle = E_n \hat{a}|n\rangle$$

$$(\hat{a}^+\hat{a} + 1/2)\hat{a}|n\rangle = (E_n - \hbar\omega)\hat{a}|n\rangle$$

The energy for $\hat{a}|n\rangle$ state is $E_n - \hbar\omega$
→ Annihilation operator

$\hat{a}^+|n\rangle$ state is at energy $E_n + \hbar\omega$
→ Creation operator.

$$\hat{a}|n\rangle = C_{n-1}|n-1\rangle$$

$$\langle n|\hat{a}^+\hat{a}|n\rangle = \langle n-1|C^*_{n-1} C_{n-1}|n-1\rangle \Rightarrow \hat{a}|n\rangle = \sqrt{n}|n-1\rangle$$

$$n = |C_{n-1}|^2 \cdot 1$$
$$C_{n-1} = \sqrt{n}$$

similarly $\hat{a}^+|n\rangle = \sqrt{n+1}|n+1\rangle$

$$\hat{a}^{+}|0\rangle = \sqrt{1}\,.\,|1\rangle$$
$$\hat{a}^{+2}|0\rangle = \sqrt{1}\,\sqrt{2}\,|2\rangle$$
$$\vdots$$
$$\hat{a}^{+n}|0\rangle = \sqrt{n!}\;|n\rangle$$

$$|n\rangle = \frac{\hat{a}^{+}|0\rangle}{\sqrt{n!}}$$

$\hat{a}|0\rangle = 0 \longrightarrow$ Null vector.

PrD

$$\langle n-1|a|n\rangle$$

$$\langle n+1|\hat{a}^{+}|n\rangle$$

H and N are hermetian operator $\hat{a}$ and $\hat{a}^{+}$ are non hermitian operator.

Photon no. state are orthogonal $\langle n|n-1\rangle = 0$

and they form a complete set

$$\sum_n |n\rangle\langle n| = \hat{I} \quad \text{identity operator}$$

Expected value of $\hat{E}_x$ & $\hat{B}_y$ for $|n\rangle = 0$

$$\hat{E}_x = E_0(\hat{a}^{+} + \hat{a})\sin kz$$

$$\langle n|\hat{E}_x|n\rangle = E_0 \sin kz \langle n|\hat{a} + \hat{a}^{+}|n\rangle$$

$$= E_0 \sin kz \left[\underbrace{\langle n|\sqrt{n}|n-1\rangle}_{=0} + \underbrace{\langle n|\sqrt{n+1}|n+1\rangle}_{=0}\right]$$

$$= \langle n|n-1\rangle = 0$$

$$\langle n|\hat{B}_y|n\rangle = 0$$

$$\hat{E}_x(z,t) = E_0\,(\hat{a}^+ e^{i\omega t} + \hat{a}\,e^{-i\omega t})\,\sin kz$$

Expected value for $\hat{E}_x^2$ & $\hat{B}_y^2$

$$\langle n|\,\hat{E}_x^2\,|n\rangle$$

$$= E_0^2 \sin^2 kz\left(\langle n|\hat{a}\hat{a} + \hat{a}^+\hat{a}^+ + \hat{a}\hat{a}^+ + \hat{a}^+\hat{a}\,|n\rangle\right)$$

$$= E_0^2 \sin^2 kz\,[1+2n]$$

Variance in $\hat{E}_x$:

$$\langle \Delta E_x\rangle^2 = \langle n|\hat{E}_x^2|n\rangle - \langle n|\hat{E}_x|n\rangle^2$$
$$= E_0^2 \sin^2 kz\,(1+2n)$$

$$\text{std. div} = E_0 \sin kz\,(1+2n)^{1/2}$$

Heisenberg uncertainity Principle

consider two operator which do not commute

$$[\hat{A}\;\hat{B}] = \hat{C}$$

Product of the stand div in the measurement of $\hat{A}$ and $\hat{B}$ are lower bound as follows

$$\Delta A\,\Delta B \geq \frac{1}{2}\,|\langle\hat{C}\rangle|$$

(look up the proof!!)

what is the product of ΔE_x & ΔN?

$$[\hat{E}_x, \hat{N}] = E_0 \sin kz [\hat{a} + \hat{a}^+, \hat{a}^+ \hat{a}]$$

$$= E_0 \sin kz \left([\hat{a}, \hat{a}^+ \hat{a}] + [\hat{a}^+, \hat{a}^+ \hat{a}] \right)$$

$$= E_0 \sin kz \, (\hat{a} - \hat{a}^+) \equiv \hat{C}$$

$$\langle \hat{C} \rangle = \langle n | E_0 \sin kz \, (\hat{a} - \hat{a}^+) | n \rangle = 0$$

$\Delta E_x \cdot \Delta N \geq 0$ for number state

minimum uncertainity in simultaneous measurement of

$\hat{E}_x$ & $\hat{N} = 0$

$$\langle \hat{C} \rangle = \langle \psi | \hat{C} | \psi \rangle$$

$$\Delta E_x \cdot \Delta N \geq \frac{1}{2} | \langle \psi | \hat{C} | \psi \rangle |$$

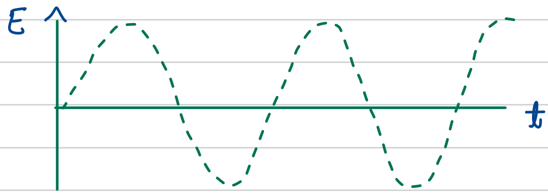
☆ This non zero error is related to simultaneous measurement wave and particle property

☆ $\Delta n \cdot \Delta \phi \geq 1$

Lecture-20

Quantum Optics (continued)

Quadrater Operators



$E = x_1 \cos\omega t + x_2 \sin\omega t$

$\downarrow$ in phase $\qquad$ $\downarrow$ quadrature phase

Coherent communicatio
Tx & detection

$$\hat{E}_x = E_0 (\hat{a} + \hat{a}^+) \sin tz$$

$$\hat{a}(t) = \hat{a}(0) e^{-i\omega t}$$

$$\hat{a}^+(t) = \hat{a}^+(0) e^{i\omega t}$$

$$\hat{E}_x(t) = E_0 (\hat{a}(0) e^{-i\omega t} + \hat{a}^+(0) e^{+i\omega t}) \sin tz$$

$$= E_0 (\hat{a}(\cos\omega t - i\sin\omega t) + \hat{a}^+(\cos\omega t + i\sin\omega t)) \sin kz$$

$$= E_0 \left(2\cos\omega t \left(\frac{\hat{a}+\hat{a}^+}{2}\right) + \frac{2}{i}\sin\omega t \left(\frac{\hat{a}+\hat{a}^+}{2}\right)\right) \sin kz$$

Quadrature operator : $\left.\begin{aligned} \hat{X}_1 &= \frac{\hat{a}+\hat{a}^+}{2} \\ \hat{X}_2 &= \frac{\hat{a}-\hat{a}^+}{2} \end{aligned}\right\}$

$$E_x(t) = 2E_0 (\hat{X}_1 \cos\omega t + \hat{X}_2 \sin\omega t) \sin kz$$

Find the expectation value for $\hat{x}_1$ & $\hat{x}_2$ number State $|n\rangle$

$$\langle n|\hat{x}_1|n\rangle = \langle n|\frac{\hat{a}+\hat{a}^+}{2}|n\rangle = 0$$

$$\langle n|\hat{x}_2|n\rangle = 0$$

find $\langle n|\hat{x}_1|n\rangle$ & $\langle n|\hat{x}_2|n\rangle$

$$\rightarrow = \frac{1}{4}\langle n|\hat{a}^2 + \hat{a}^{+2} + \hat{a}\hat{a}^+ + \hat{a}^+ a|n\rangle$$

$$= \frac{1}{4}\langle n|1+2a^+a|n\rangle$$

$$= \frac{1}{4}[2n+1]$$

$$\langle n|\hat{x}_2^2|n\rangle = -\frac{1}{4}\langle n|\hat{a}^{+2} + \hat{a}^2 - \hat{a}\hat{a}^+ - \hat{a}^+\hat{a}|n\rangle$$

$$= \frac{1}{4}(2n+1)$$

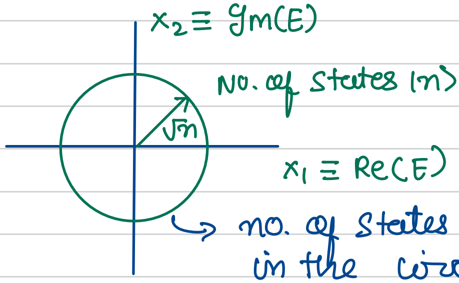
Variance of the quadrature operators

$$\langle \Delta\hat{x}_1^2\rangle = \langle n|\hat{x}_1^2|n\rangle - (\langle n|x_1|n\rangle)^2 = \frac{1}{4}(2n+1)$$

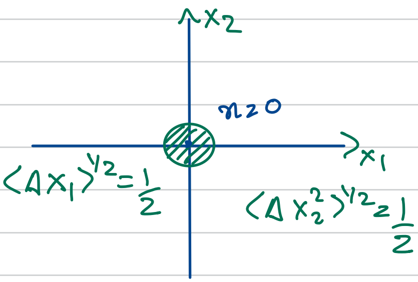
$$\langle \Delta\hat{x}_2^2\rangle = \frac{1}{4}(2n+1)$$

For Vacuum State we get min value of variance for $\hat{x}_1$ & $\hat{x}_2$ minimum uncertainity state.

Phase Space representation of the number States



↳ no. of states can be located anywhere in the circumference of the circle



Phase space representation of Vaccum state

These pictures are qualitative in nature, good for visualization of noise and the location of the state

As quadrature states have no classical analogy this should not be taken too literally.

Coherent state

States which give the most sensible representation of classical states (laser source)

↳ Representation of the electric field evolving in space and time

$$E(z,t) = E_0 \cos(\omega t - kz)$$

Expectation values is a non zero quantity.

↳ Noise limited by shot-noise

↳ calculated the uncertainity for $\hat{X}_1$ & $\hat{X}_2$.

$$|\psi\rangle = |n\rangle \Rightarrow E(\hat{E}_x) = E(\hat{X}_1) = E(\hat{X}_2) = 0$$

$$|\psi'\rangle = C_n|n\rangle + C_{n+1}|n+1\rangle$$

ensures $E(\hat{E}_x) \neq 0$

$$|\alpha\rangle = \sum_{n=0}^{\infty} C_n|n\rangle$$

Consider the eigenvalue equation for $\hat{a}$:

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

$\alpha \rightarrow$ Eigen Value

$|\alpha\rangle \rightarrow$ Eigen state

$$\langle\alpha|\hat{a}^{+} = \alpha^{+}\langle\alpha|$$

$$\hat{a}\sum_{n=0}^{\infty} C_n|n\rangle = \alpha \sum_{n=0}^{\infty} C_n|n\rangle$$

$$\sum_{n=1}^{\infty} C_n\sqrt{n}\,|n-1\rangle = \alpha\sum_{n=0}^{\infty} C_n|n\rangle$$

$$\sum_{n=1}^{\infty} C_n\sqrt{n}\,|n-1\rangle = \alpha\sum_{n=0}^{\infty} C_n|n\rangle \qquad n-1=m$$

$$\sum_{m=0}^{\infty} C_{m+1}\sqrt{m+1}\,|m\rangle = \alpha\sum_{m=0}^{\infty} C_n|n\rangle$$

$$\alpha C_n = C_{n+1}\sqrt{n+1} \longrightarrow \text{Recursive relationship}$$

$$C_n = \frac{\alpha C_{n-1}}{\sqrt{n}} = \frac{\alpha}{\sqrt{n}} \cdot \frac{\alpha}{\sqrt{n-1}} C_{n-2} = \frac{\alpha}{\sqrt{n}} \frac{\alpha}{\sqrt{n-1}} \cdots \frac{\alpha C_0}{\sqrt{1}} = \frac{\alpha^n C_0}{\sqrt{n!}}$$

Relating C_n to C_0

$$\langle\alpha|\alpha\rangle = 1$$

$$|\alpha\rangle = \sum_{n=0}^{\infty} C_0 \frac{\alpha^n}{\sqrt{n!}}$$

Note : α can be complex no.

$$= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \frac{C_0^* \alpha^{*i}}{\sqrt{i!}} \cdots \frac{C_0 \alpha^j}{\sqrt{j!}} \langle i|j\rangle$$

$$\langle i|j\rangle = \delta_{ij}$$

$$= \sum_{i=0}^{\infty} \frac{|C_0|^2 |\alpha|^{2i}}{i!} = |C_0|^2 \sum_{i=0}^{\infty} \frac{|\alpha|^{2i}}{i!} = |C_0|^2 e^{+|\alpha|^2} = 1$$

$$C_0 = e^{-|\alpha|^2/2}$$

$$|\alpha\rangle = \sum_{n=0}^{\infty} \frac{e^{-|\alpha|^2/2}}{\sqrt{n!}} \alpha^n |n\rangle$$

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

$$\langle\alpha|\hat{a}^+ = \alpha^* \langle\alpha|$$

What is the expectation value of $\hat{E}_x$?

$\langle\alpha|\hat{E}_x|\alpha\rangle = ?$ for propogating plane wave

$$\hat{E}_x = E_0 (\hat{a} e^{-i\omega t} e^{-ikz} + \hat{a}^+ e^{i\omega t} e^{+ikz})$$

This is slightly different from the standing wave $\hat{E}_x$ represented in the optical cavity

$$= E_0 \langle \alpha | \hat{a} e^{-i(\omega t + kz)} + \hat{a}^+ e^{+i(\omega t + kz)} | \alpha \rangle$$

$$= E_0 \left[\alpha e^{-i(\omega t + kz)} + \alpha^* e^{+i(\omega t + kz)} \right]$$

$$\alpha = |\alpha| e^{i\theta}$$

$= 2E_0 |\alpha| \cos(\omega t + kz + \theta) \Rightarrow$ non zero expectation value which is similar to classical E field evolving in space and time

$$\langle \alpha | E_x^2 | \alpha \rangle$$

$$= E_0^2 \langle \alpha | \hat{a}^2 e^{-2i(\omega t + kz)} + \hat{a}^{+2} e^{+2i(\omega t + kz)} + \hat{a}\hat{a}^+ + \hat{a}^+\hat{a} | \alpha \rangle$$

$$= E_0^2 \left[\alpha^2 e^{-i(\omega t + kz)} + \alpha^{*2} e^{2i(\omega t + kz)} + 1 + \langle \alpha | 2\hat{a}^+\hat{a} | \alpha \rangle \right]$$

$$= E_0^2 \underbrace{\left[1 + 2|\alpha|^2 \cos(2\omega t + 2kz + 2\theta) + 2|\alpha|^2 \right)}_{4|\alpha|^2 \cos^2(\omega t + kz + \theta)}$$

$$\langle \Delta \hat{E}_x^2 \rangle = \langle \alpha | \hat{E}_x^2 | \alpha \rangle - \langle \alpha | \hat{E}_x | \alpha \rangle^2 = E_0^2$$

$\langle \alpha | \hat{N} | \alpha \rangle = \langle \alpha | \hat{a}^+ \hat{a} | \alpha \rangle = |\alpha|^2 = N \rightarrow$ Mean no. of photons

$$\langle \alpha | \hat{N} | \alpha \rangle = \langle \alpha | \hat{a}^+ \hat{a} \hat{a}^+ \hat{a} | \alpha \rangle = |\alpha|^2 \langle \alpha | \hat{a} \hat{a}^+ | \alpha \rangle$$

$$= |\alpha|^2 \langle \alpha | 1 + \hat{a}^+ \hat{a} | \alpha \rangle$$

$$= |\alpha|^2 \left[|\alpha|^2 + 1 \right]$$

$$= N[N+1]$$

$\langle \Delta \hat{N}^2 \rangle = N(N+1) - N^2 = N$

Mean and variance are identical $= N$

Q what is the probability of measuring n photon per a instance of the measurement?

$$P(n) = |\langle n|\alpha\rangle|^2 = \frac{e^{-|\alpha|^2}|\alpha|^{2n}}{n!} = \frac{e^{-N}N^n}{n!}$$

which is the poisson distribution

* Signal to noise ratio (SNR)

$$\frac{\text{Mean}^2}{\text{variance}} \sim \frac{\text{Mean}}{\text{std.dev}}$$

$SNR = \frac{N^2}{N} = N$, $\frac{N}{\sqrt{N}} = \sqrt{N} \Rightarrow$ quantum limit of measurement uncertainity shot noise limited uncertainity.

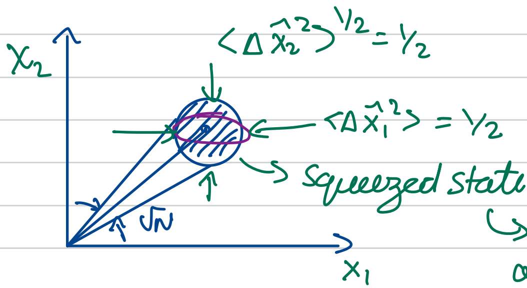
Relative uncertainity $\frac{1}{N}$ or $\frac{1}{\sqrt{N}}$

Squeezed state of light achieve sub-shot noise performance.

$$\hat{X}_1 = \frac{\hat{a} + \hat{a}^+}{2} \qquad \hat{X}_2 = \frac{\hat{a} - \hat{a}^+}{2i}$$

$\left.\langle \Delta \hat{X}_1^2 \rangle = 1/4, \langle \Delta \hat{X}_2^2 \rangle = 1/4 \right\}$ The variance in quadrature measurement of coherent states is equal to the vaccum state.

Phase Space representation of coherent states



↳ less shot noise for $\hat{X}_2$ and more than shot noise for $\hat{X}_1$

* Displacement operator $\hat{D}$

$$\hat{D}|0\rangle = |\alpha\rangle$$

$$\hat{D}(\alpha) = e^{\alpha \hat{a}^{+} - \alpha^{*}\hat{a}}$$

Refer to the proof in gerry and knight section 3.2

Phase shifting Operator

$$\hat{U}(\theta) = e^{-i\theta\hat{N}}$$

$$U(\theta)|n\rangle = \underbrace{e^{-i\theta\hat{N}}}|n\rangle$$

expanding this exponential

$$= e^{-i\theta n}|n\rangle \Rightarrow$$ no. state gets phase shifted by $n\theta$.

$\hat{U}(\theta)|\alpha\rangle = |\alpha e^{i\theta}\rangle \longrightarrow$ coherent state gets phase shifted by θ.

Refer to gerry and knight section 3.2